Compactness and non-compactness theorems of the constant Q_4 and Q_6 -curvature problems

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We provide a complete resolution to the question of compactness for the full solution sets of the fourth-order and sixth-order constant Q-curvature problems on smooth closed Riemannian manifolds.

Firstly, we prove that the solution set of the fourth-order constant Q-curvature problem is C^4 -compact in dimensions $5 \leq n \leq 24$.

For $n \geq 25$, an example of an L^∞-unbounded sequence of solutions has been known for over a decade (Wei and Zhao (2013)).

Additionally, compactness has been established for $5 \leq n \leq 9$ by Li and Xiong (2019).

Secondly, we demonstrate that the solution set of the sixth-order constant Q-curvature problem is C^6-compact in dimensions $7 \leq n \leq 26$, while a blow-up example exists for $n \geq 27$.

Our principal observation is that the linearized equations associated with the Q-curvature problems can be transformed into overdetermined linear systems, which admit nontrivial solutions due to unexpected algebraic structures of the Paneitz operator and the sixth-order GJMS operator.

This key insight not only plays a crucial role in deducing the compactness result for high dimensional manifolds,but also reveals an elegant hierarchical pattern with respect to the order of conformally covariant operators, suggesting the possibility of a unified theory of the compactness of the constant Q-curvature problems of all orders. (Joint work with Liuwei Gong and S. Kim.)

Gap theorem for nonnegatively curved manifolds

Pak Yeung Chan(陳柏揚)
Tamkang University

In this talk, we shall discuss some recent results on the gap theorem of nonnegatively curved manifolds with small curvature in an average integral sense, which can be viewed as a Riemannian analog of the optimal gap result by Ni on Kahler manifolds. In dimension 3, we also establish a gap theorem for Ricci nonnegative manifolds with pointwise quadratic curvature decay and fast average integral curvature decay. This talk is based on some joint works with Man-Chun Lee.

CR invariant surfaces and hyperbolic equations

Jih-Hsin Cheng(鄭日新)
Academia Sinica

We introduce two CR invariant surface energies E_1 and E_2discovered in mid-nineties and express them in terms of quantities in pseudohermitian geometry. The E_2-energy appears to be the log term

coefficient in the expansion of the volume renormalization. We study the E_1-minimizers while (nonnegative) E_1 is an analogue to the Willmore energy in conformal geometry. Surprisingly the equation $E_1 = 0$ is hyperbolic. We solve an initial-value problem via the principle of bicharacteristic curves and classify all the solutions of rotational invariance in the Heisenberg group. This presentation includes joint works with Paul Yang-Yongbing Zhang and Hung-Lin Chiu respectively.

Expanding Ricci solitons asymptotic to cones with nonnegative scalar curvature

Eric Chen
University of Illinois Urbana-Champaign

In dimensions four and higher, the Ricci flow may encounter singularities modelled on cones with nonnegative scalar curvature. It may be possible to resolve such singularities and continue the flow using expanding Ricci solitons asymptotic to these cones, if they exist. I will discuss joint work with Richard Bamler in which we develop a degree theory for four-dimensional asymptotically conical expanding Ricci solitons, which in particular implies the existence of expanders asymptotic to a large class of cones.

Rigidity Theorem for Poincare-Einstein Manifolds

Fang Wang(王芳)
Shanghai Jiao Tong University

The rigidity problem for Poincare-Einstein manifold asks: when the conformal infinity of a Poincare-Einstein manifold (X, g) is the standard round sphere or Euclidean space, is (X, g) the standard hyperbolic space? In this talk, I will first introduce the classical rigidity theorem, under the condition that (X, g) is C^3 conformally compact. Then I will report some recent rigidity result for Poincare-Einstein manifold in the upper half-plane model, which takes the Euclidean space as the conformal infinity and whose adapted conformal metric has quadratic curvature decay at infinity. This is joint work with Sanghoon Lee (KIAS).

Harmonic maps from the product of hyperbolic spaces to hyperbolic spaces

Kazuo Akutagawa(芥川 和雄)
Chuo University

In this talk, we will consider the asymptotic Dirichlet problem for harmonic maps from the product $H^{m_1} \times H^{m_2}$ of two hyperbolic spaces to hyperbolic spaces. It remarks that $H^{m_1} \times H^{m_2}$ is a higher rank symmetric space of noncompact type. We first show uniqueness and nonexistence results, particularly the existence of such harmonic maps (with some natural conditions) implies that it must

be $m_1 = m_2 = 2$. We also show an existence result for harmonic maps from $H^2 \times H^2$ to hyperbolic spaces. This is a joint work with Yoshihiko Matsumoto.

Existence of fully nonlinear Yamabe metrics on noncompact manifolds

Yi Wang(王一)
Johns Hopkins University

In this talk, we are going to talk about existence results for a class of fully nonlinear Yamabe problems on noncompact manifolds in positive and negative cones. We will also discuss examples of manifolds with asymptotically flat ends that satisfy the hypotheses of our theorems. This is joint work with Jonah Duncan.

Yang-Mills Theory on Conformally Compact Manifolds

Andrew Waldron
University of California, Davis

The Yang-Mills equations are central to the study of smooth 4-manifolds and particle interactions. We consider Yang-Mills theory in the setting of conformally compact manifolds in general dimensions. In particular, we obtain a formula for the renormalized energy of Yang-Mills solutions on Poincaré-Einstein 6-manifolds. The method generalizes the proof of Chang-Qing-Yang for renormalized volumes to a broader setting.

Overdetermined boundary value problems in a Riemannian manifold

Keomkyo Seo
Sookmyung Women's University

Serrin's overdetermined problem is a famous result in mathematics that deals with the uniqueness and symmetry of solutions to certain boundary value problems. It is called "overdetermined" because it has more boundary conditions than usually required to determine a solution, which leads to strong restrictions on the shape of the domain. In this talk, we discuss overdetermined boundary value problems in a Riemannian manifold and discuss a Serrin-type symmetry result to the solution to an overdetermined Steklov eigenvalue problem on a domain in a Riemannian manifold with nonnegative Ricci curvature and it will be discussed about an overdetermined problems with a nonconstant Neumann boundary condition in a warped product manifold.

Backward Uniqueness of Extrinsic Geometric Flow

John Man Shun Ma(马文信)
Southern University of Science and Technology

n this talk, we discuss a backward uniqueness theorem for extrinsic geometric flow of possibly non-compact hypersurfaces in general ambient complete Riemannian manifolds. The theorem is applicable to a wide range of extrinsic geometric flow, including the mean curvature flow, inverse mean curvature flow, Gauss curvature flow and so on. This is a joint work with Dasong Li.

CR Paneitz operator and embeddability

Yuya Takeuchi(竹内有哉)
University of Tsukuba

The CR Paneitz operator, a CR invariant fourth-order linear differential operator, plays a crucial role in three-dimensional CR geometry. It is deeply connected to global embeddability, the CR positive mass theorem, and the logarithmic singularity of the Szegő kernel. In this talk, I will discuss recent progress on the spectrum of the CR Paneitz operator. Specifically, I will focus on differences in its nature depending on whether it is embeddable or not.

The minimizers of a CR invariant energy functional E_1 in the Heisenberg group H_1.

Hung-Lin Chiu(邱鴻麟)
National Tsing Hua University

In early 90 's, J.-H. Cheng discovered two CR invariant surface elements dA_1 and dA_2, via Cartan-Chern's method of admissible frames. Later, in 2018, J.-H. Cheng, P. Yang and Y. Zhang showed that for a nonsingular surface $\Sigma \subset M$, they can express dA_1 and dA_2 by quantities in pseudohermitian structure, and thus defined two CR invariant energy functionals E1 and E2. In this talk, we will show that, in the Heisenberg group, there are only four classes of E1-minimizers in the category of rotationally symmetry. We will also give a uniqueness theorem for minimizers with the same initial values on a non-characteristic curves.

Positvity of Q-curvature

Xingwang Xu(徐兴旺)
Nanjing University

Q-curvature is the geometric quantity associated to the higher order Paneitz-Branson operator. In flat case, it is just a poly-harmonic operator.In general, the maximum principle does not work for such differential operator. In this talk, I should report, in very simple case, we can ensure the maximum principle by considering the geometric information. This is a joint work with M. Li.

A positive mass theorem for non-smooth metrics on asymptotically flat manifolds with non-compact boundary

Sergio Almaraz
Universidade Federal Fluminense

On a smooth asymptotically flat Riemannian manifold with non-compact boundary, we present a positive mass theorem for metrics which are only continuous across a compact hypersurface. As an application, we obtain a positive mass theorem on manifolds with non-compact corners. This is a joint work with Shaodong Wang.

Infinitely many non-radial sign-changing solutions for critical Hamiltonian systems in the Euclidean space

Seunghyeok Kim(金升赫)
Hanyang University

We build infinitely many geometrically distinct non-radial sign-changing solutions for the Lane-Emden systems in the Euclidean space. These systems can be regarded as generalizations of the Yamabe problem in the Euclidean space (or on the unit sphere) to Hamiltonian-type elliptic systems. A key feature of these systems is that they are energy-critical (the associated energy is invariant under the translation and natural dilations), but they are not Kelvin invariant (not invariant under sphere inversions). To establish this result, we introduce several new ideas and strategies that are both robust and potentially applicable to other critical problems lacking the Kelvin invariance. This is a joint work with Yuxia Guo (Tsinghua University, P. R. China), Angela Pistoia (Sapienza Università di Roma, Italy), and Shusen Yan (Central China Normal University, P. R. China).

On the Gauss-Bonnet-Chern formula on Poicaré-Einstein manifolds

Wei Yuan(袁伟)
Sun Yat-sen University

Gauss-Bonnet-Chern formula is a remarkably fundamental result which builds a connection between differential geometry and topology. It has been successfully generated on Poicare-Einstein manifolds through renormalized curvature integral by Albin. With the aid of ambient space construction, we give a general formulism for renormalized curvature integral. In particular, we give a reformulation of Guass-Bonnet-Chern formula on Poicare-Einstein manifolds, which provides some interesting applications. This work is a joint work with Jeffrey S. Case, Ayush Khaitan, Yueh-Ju Lin and Aaron J. Tyrrell.

Geometry of the Liouville equation

Mijia Lai(来米加)
Shanghai Jiao Tong University

The Liouville equation in the plane is given by $\Delta u + e^{2u} = 0$. In this presentation, I will provide an overview of certain geometric aspects of this widely studied equation.

Revisiting Modica's estimates for fourth-order Hénon equations in the whole space

Quoc Anh Ngo
Vietnam National University

Forty years ago, via the maximum principle, L. Modica established a gradient estimate for bounded entire solutions to the nonlinear second-order equation $\Delta u = F'(u)$ in the Euclidean space. It turns out that a Modica type estimate for fourth-order Hénon equations is also available, despite the lack of maximum principles. Ten years ago, this higher-order analogous estimate was strengthened in a work of M. Fazly, J. Wei, and X. Xu by incorporating a gradient term, leading to several interesting applications in conformal geometry. This talk focuses on the improved inequality introduced by Fazly, Wei, and Xu. Specifically, I will demonstrate that the enhanced inequality remains valid for a broader range of parameters, and it holds for any solution, not necessarily bounded. In certain cases, the inequality is in fact sharp.

Recent progress in Liouville theorems and removable singularities

Zongyuan Li(李宗元)
City University of Hong Kong

In this talk, I will discuss recent advances in Liouville-type theorems for conformally invariant, fully nonlinear elliptic equations. A key step in these results is the development of a removable singularity theory for solutions with isolated boundary or interior singularities. This is based on joint work with B. Z. Chu and Y. Y. Li (Rutgers).